Exercise 3

(a) Find y' by implicit differentiation.

(b) Solve the equation explicitly for y and differentiate to get y' in terms of x.

(c) Check that your solutions to parts (a) and (b) are consistent by substituting the expression for y into your solution for part (a).

$$\sqrt{x} + \sqrt{y} = 1$$

Solution

Part (a)

Differentiate both sides with respect to x.

$$\frac{d}{dx}(\sqrt{x} + \sqrt{y}) = \frac{d}{dx}(1)$$

$$\frac{d}{dx}(\sqrt{x}) + \frac{d}{dx}(\sqrt{y}) = 0$$

$$\frac{1}{2}x^{-1/2} + \frac{1}{2}y^{-1/2} \cdot \frac{d}{dx}(y) = 0$$

$$\frac{1}{2\sqrt{x}} + \frac{y'}{2\sqrt{y}} = 0$$

Solve for y'.

$$y' = -\sqrt{\frac{y}{x}}$$

Part (b)

Solve for y first.

$$\sqrt{y} = 1 - \sqrt{x}$$

$$y = (1 - \sqrt{x})^2 = 1 - 2\sqrt{x} + x$$

Then take the derivative.

$$y' = \frac{d}{dx}(1 - 2\sqrt{x} + x)$$

$$= -2 \cdot \frac{1}{2}x^{-1/2} + 1$$

$$= 1 - \frac{1}{\sqrt{x}}$$

Plug the formula for $\sqrt{y}$ into the result of part (a) to see if the same answer is obtained.

$$y' = -\frac{\sqrt{y}}{\sqrt{x}} = -\frac{1 - \sqrt{x}}{\sqrt{x}} = 1 - \frac{1}{\sqrt{x}}$$